Reliability Analysis of Aircraft Structures under Random Loading and Periodic Inspection

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A reliability analysis of fatigue-sensitive aircraft structures, based on the application of the approach developed in the "random vibration theory," is presented. Operational service loads, composed of ground loads, ground-air-ground loads, and gust loads, are all random in nature. The fatigue process involved here consists of crack initiation, crack propagation, and strength degradation. The time to crack initiation and the ultimate strength are random variables. After a fatigue crack is initiated, fracture mechanics is applied to predict crack propagation under random loading. While the fatigue crack is propagating, the residual strength of the structure decreases progressively, thus increasing the failure rate with time. The aircraft structure is subjected to periodic inspection in service. When a fatigue crack is detected during inspection, the implicated component is either repaired or replaced, so that both the static and the fatigue strength are renewed. Such a renewal process is taken into account in the present analysis. The detection of an existing fatigue crack during inspection is also a random variable which depends on the resolution capability of the particular technique employed and the size of the existing crack. Taking into account all the random variables as well as all the random loadings, the solution for the probability of first failure in a fleet of aircraft is derived. Finally, numerical examples are given to demonstrate the effect of inspection and fleet size on the fleet reliability.

I. Introduction

FATIGUE has been a problem in the design of many structures in mechanical engineering (e.g., turbine blades, propeller shafts); in aeronautical engineering (e.g., aircraft structures), and in civil engineering (e.g., buildings, highway and railroad bridges, etc.). The problem of fatigue is further complicated by random nature of most of the loading inputs to these structures in service.[1-6] Typical examples are gust and maneuver loads on aircraft,[7-13] wind and earthquake forces on buildings, and traffic loading to bridges, to mention just a few.

Fatigue damage is revealed in a structure by the initiation of a visible crack. It has been a practice, for example, on railroad bridges and aircraft structures, to periodically inspect fatigue-sensitive structures in order to detect such cracks and to repair or replace the cracked components.[14-17] Inspection is an important procedure to increase the reliability of fatigue-critical structures. Hence, reliability analysis of fatigue-sensitive structures, under random loading and periodic inspection, is of practical importance, and is the primary concern of this study. Although the application of reliability analysis to aircraft structures is emphasized, the approach discussed in this paper is equally applicable to other fatigue-sensitive structures such as civil and mechanical engineering structures, under random loading.

The specific type of random loading considered herein is a flight-by-flight loading to transport-type aircraft (bombers, tankers, etc.). It consists of ground loads, ground-air-ground loads, and gust loads, which are all random in nature. The ultimate strength of the structure is a random variable with certain statistical variability.[18,19] Failure occurs as soon as the strength, either the ultimate strength or the residual strength after crack initiation, is exceeded by the random load level. This is referred to as the first-passage or first-excursion failure in random vibration.[20-23]

The fatigue process considered consists of 1) crack initiation, 2) crack propagation, and 3) strength degradation. The time to crack initiation is a random variable and is assumed to have a two-parameter Weibull distribution.[24,25] After the fatigue crack is initiated, fracture mechanics is applied to estimate crack propagation under random loading, where the statistics of rise and fall of random loading plays an important role.[26-29] While the crack is propagating, the ultimate strength is reduced progressively. As a result, the residual strength of a cracked structure decreases, thus increasing the failure rate (or risk function) in time.[30]

The residual strength after crack initiation is related either to the ultimate strength and the crack size through the Griffith-Irwin equation for nonredundant structures[31,32] or is determined by testing and analysis for redundant structures.[14,15,33-36] With the concept of fail-safe design, fatigue crack propagation will be arrested by the "crack stopper"; thus the fail-safe crack size defines the maximum crack allowable in the structure.

The inspection is performed at periodic intervals to detect the fatigue crack if it exists. When a crack is detected, the cracked component is repaired or replaced so that both the residual strength and the fatigue strength of the component are renewed. This renewal process is taken into account in the present reliability analysis. During inspection, however, the fatigue crack may not be detected. The detection of an existing crack is also a random variable, which depends on the resolution capability of a particular method or technique employed for inspection. The probability of crack detection, in general, is an increasing function of the existing crack size.[32,37]

Taking into account all the random variables and random loadings described above, the solution for the probability of failure is derived through application of the conditional probability theory. Numerical results are given to demonstrate the significant influence of the inspection frequency and the fleet size on the fleet reliability. Further studies needed in the area of aircraft structural design based on the reliability philosophy are discussed in detail.

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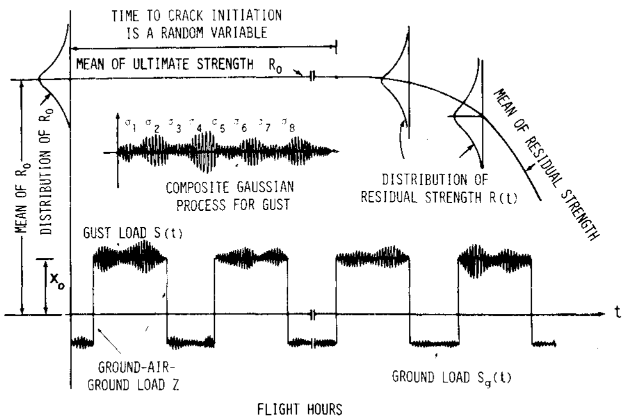

Fig. 1 Profile by flight-by-flight load spectrum, ultimate strength, and residual strength.

II. Random Loading

Consider a designed flight-by-flight random loading history (see Fig. 1), where each flight has three different characteristics: 1) ground loads $S_g(t)$, 2) gust loads $S(t)$, and 3) ground-air-ground loads Z. This specific type of random loading has been used for the design of transport-type aircraft (bombers, logistic aircraft, etc.).

The ground loads $S_g(t)$, resulting from landing and takeoff, have been modeled as a random process.[1] They produce compressive stresses in certain fatigue-critical locations (i.e., wing lower surface), and have some effect on the fatigue life. It has been observed in fatigue experiments, that when the general loading range of a specimen is in tension, the introduction of occasional high-level loads results in a prolongation of the fatigue life due to the effect of beneficial residual stresses. This beneficial effect, however, may be reduced or even eliminated when compressive stresses are introduced in the loading history under certain conditions.

In each flight, there is one cycle of ground-air-ground load Z, which is also a random variable over the life of the aircraft. The magnitude or range of this load cycle relative to the flight loads may be large enough to have a profound effect on the fatigue life of the structure.

A catastrophic failure of the structure of transport aircraft is essentially due to gust loading, since failure occurs when the ultimate strength (or the residual strength after crack initiation) is exceeded. The gust loading $S(t)$, modeled as a stationary composite Gaussian process,[7-13] will be adopted herein and is described briefly in the following:

The gust loading $S(t)$ consists of a series of turbulence patches modeled as stationary Gaussian random processes $S(t, \sigma_i)$, $i = 1, 2, \ldots$, where σ_i is the standard deviation. The power spectral densities $G_i(\omega)$ for $S(t, \sigma_i)$, $i = 1, 2 \ldots$ are identical when normalized with respect to σ_i^2, i.e., $G_i(\omega)/\sigma_i^2$ is invariant for all $i = 1, 2, \ldots$.

The expected number of upcrossings (or upcrossing rate) per load cycle $v^+(R_0, \sigma_i)$ across a threshold R_0, the ultimate strength, by $S(t, \sigma_i)$ is well-known[1]

$$v^+(R_0, \sigma_i) = \exp[-(R_0 - X_0)^2/2\sigma_i^2] \tag{1}$$

where X_0 is the average value of $S(t, \sigma_i)$, which is equal to the stress associated with one g loading (see Fig. 1). The standard deviations σ_i, $i = 1, 2 \ldots$ are assumed to be statistically independent and identically distributed random variables with a half-normal distribution[12,13]

$$f_\sigma(x) = P_1(2/\pi\sigma_{c1}^2)^{-1/2} \exp(-x^2/2\sigma_{c1}^2) + P_2(2/\pi\sigma_{c2}^2)^{-1/2} \exp(-x^2/2\sigma_{c2}^2) \tag{2}$$

where $f_\sigma(x)$ is the density function of σ_i, and P_1 and P_2 represent the fractions of nonstorm turbulence (clear air) and the thunderstorm turbulence, respectively, with associated intensities σ_{c1} and σ_{c2}. Hence, $P_1 + P_2 = 1$. Parameters P_1, P_2, σ_{c1}/B and σ_{c2}/B are referred to as turbulence field parameters and are specified in Ref. 7 for various altitudes, where B represents the structural characteristics.[12] A unified approach for the determination of these parameters from the measured turbulence data has recently been proposed in Refs. 12 and 13. The average number of upcrossings per cycle (one cycle is defined as one upcrossing of the mean X_0) by the process $S(t)$ is obtained therefore as

$$v^+(R_0) = \int_0^\infty v^+(R_0, x) f_\sigma(x)\, dx = P_1 e^{-(R_0 - X_0)/\sigma_{c1}} + P_2 e^{-(R_0 - X_0)/\sigma_{c2}} \tag{3}$$

Thus the upcrossing rate of a threshold R_0 is an exponential function. This has been verified by extensive turbulence field data,[7-13] and Eq. (3) has been used in the current U.S. Air Force specification[7] for aircraft structural design for atmospheric turbulence.[12,13] The gust process $S(t)$ thus defined is referred to as a composite Gaussian process.

It should be mentioned that the first term in Eq. (3) represents the contribution from clear-air turbulence and $P_1 \gg P_2$, $\sigma_{c2} > \sigma_{c1}$. Therefore, it is primarily responsible for the fatigue initiation and crack propagation. In the current practice in random fatigue testing, the second term is usually disregarded.[15,39] The second term, representing the contribution from storm turbulence with large intensity σ_{c2} is primarily responsible for the excursion or exceedance of the ultimate strength or the residual strength of the structure. Equation (3) will be used later for computing the failure rate (or risk function) in order to estimate the structural reliability.

We digress here to comment that the fact that this turbulence model results in an exponential exceedance [Eq. (3)], does not imply that it is the correct model. This is very important, since other models of random processes may also produce an exponential exceedance such as Eq. (3). The model is employed herein for expediences in view of the fact that no simpler model for gust, in the sense for implementation in design, exists in the literature. An exploratory nonstationary model for gust loading, recently proposed by Lin[2,3] should be mentioned.

III. Material/Structural Performance and Inspection

A. Fatigue Crack Initiation

When a structure is subjected to cyclic loading for some time, a fatigue crack will be initiated first. This initiated crack will propagate progressively until a critical crack size is reached and fracture occurs. It is well known that the fatigue process consists of crack initiation, crack propagation, and final fracture. It has been shown that the statistical distribution of the time T_I to fatigue crack initiation for the critical components of aircraft structures may be represented by a two-parameter Weibull distribution.[15,24]

$$W(t) = \alpha\beta^{-1}(t/\beta)^{\alpha-1} e^{-(t/\beta)^\alpha} \tag{4}$$

where α is the shape parameter and β the scale parameter. Parameters α and β are estimated from the test results of the coupon specimens and the full-scale structure, respectively, under flight-by-flight loading shown in Fig. 1.[15,38,39] If the result of the full-scale test is not available, an alternate approach is to estimate the parameter β by use of the design mission spectrum, the pertinent S-N diagram and the cumulative damage hypothesis.

B. Crack Propagation under Random Loading

Once the fatigue crack is initiated and has a detectable size, say 0.02 in., fracture mechanics can be applied to predict the crack propagation under random loading. The applicability of fracture mechanics requires that the crack size should be large compared to the plastic zone at the crack tip. For most materials, such as aluminum, this requirement is satisfied for a detectable crack size, say 0.02 in.[32] Therefore, the power law of crack propagation under Gaussian random loading, which has been verified experimentally,[26–28] will be used:

$$da/dn = c\overline{\Delta K^b} \tag{5}$$

where a is the crack size, da/dn is the rate of crack propagation per cycle, ΔK is the range of stress intensity factor, and b and c are material constant. Equation (5) is an empirical expression in which possible retardation effects are implicitly contained. $\overline{\Delta K^b}$ is the average of the bth power of the stress intensity factor range. For aluminum under random loading, $b = 4$ seems to be appropriate.[27,28] For the sake of simplicity of presentation, we shall set $b = 4$, realizing that when b is different from 4 for other materials, the approach discussed herein remains valid and it does not involve any difficulty to account for it. Hence[26]

$$\overline{\Delta K^4} = \overline{S^4}a^2 \tag{6}$$

where $\overline{S^4}$ is the average of the fourth power of the rise and fall of the composite Gaussian process $S(t)$. Approximate methods for estimating $\overline{S^4}$ from the power spectral density are available in Refs. 26–29, and are summarized in the Appendix. Thus

$$da/dn = c\overline{S^4}a^2 \tag{7}$$

Integrating Eq.. (7) from the initial crack size a_0 to the crack size $a(t)$, after t flight hours, one obtains

$$a(t) = a_0/[1-ta_0cQ]$$
$$Q = N_0[\overline{S^4}+\overline{Z^4}N_a^{-1}+\overline{S_g^4}N_gN_a^{-1}] \tag{8}$$

in which N_0 is the number of gust load cycles per flight hour; $\overline{Z^4}$ and $\overline{S_g^4}$ are the average of the fourth power of the ground-air-ground stress cycle and the ground load, respectively. N_a and N_g are the number of gust load and ground load cycles per flight, respectively. In Eq. (8), the contribution to the crack propagation from the gust load $S(t)$, the ground load $S_g(t)$ and the ground-air-ground cycle Z have been taken into account.

C. Residual Strength

After a fatigue crack is initiated in the structure, the ultimate strength decreases because of the presence of the crack. Based on fracture mechanics, the relationship between the residual strength R of a structure containing a crack and the crack size a is given by the Griffith-Irwin equation

$$K_c = R(\pi a/2)^{1/2} \tag{9}$$

in which K_c is the critical stress intensity factor, which is a material constant. The relationship, Eq. (9), holds up to the point where R is equal to the ultimate strength R_0. As a result, there is a critical crack size a_c beyond which the strength R_0 starts to decrease following Eq. (9). This critical crack size a_c is a very important parameter in selecting or comparing candidate materials for a particular structure.[32]

Let t_c be the time (flight hours) required to reach a_c after crack initiation. Then, it follows from Eq. (8) that

$$t_c = [1-(a_0/a_c)]/ca_0Q \tag{10}$$

Let $R(t_m)$ be the residual strength at t_m flight hours after a_c; i.e., the residual strength at $t = t_c+t_m$ after crack initiation. Then, integrating Eq. (7) from a_c to $a(t_m)$, that is the crack size associated with the residual strength $R(t_m)$, and using Eq. (9), one obtains

$$R(t_m) = R_0[1-a_c cQt_m]^{1/2} \tag{11}$$

in which R_0 is the ultimate strength (see Fig. 1).

In order to prevent the crack from propagating to a catastrophic size, it has been a design practice to provide crack stoppers in the structure, which will arrest the crack. This practice is called "fail-safe design." If a_s denotes the distance between adjacent fail-safe stoppers, then it is the maximum crack size allowable in the structure, and the minimum residual strength at this crack size can be obtained from Eq. (9).

Thus far, the residual strength of a cracked structure is obtained from the Griffith-Irwin equation [Eq. (9)], which applies to nonredundant structures. Many structures, however, are designed with high redundancy. Under this circumstance, the residual strength of the cracked structure no longer follows Eq. (9), but depends on the particular design and has to be obtained by analysis and testing.[14,15,33–36] As a result, it is not possible to discuss the residual strength of a highly redundant cracked structure in general. However, the general trend is for the residual strength to be a monotonically decreasing function of the flight hours or the crack size.

Let $R_0\xi$ be the residual strength at the fail-safe crack size a_s which is determined from analysis and testing. In view of the form of Eqs. (9) and (11) as well as the test results[14,15,33–36] a possible model for the residual strength $R(t_n)$ at t_n flight hours after crack initiation is suggested as follows:

$$R(t_n) = R_0\left\{1-(1-\xi)\left(\frac{a(t_n)-a_0}{a_s-a_0}\right)^{1/2}\right\} \tag{12}$$

where $a(t_n)$ is the crack size at t_n and is computed from Eq. (8).

D. Periodic Inspection and Crack Detection

In the preceding section, the fatigue damage is expressed in terms of the fatigue crack size $a(t_n)$ [Eq. (8)], which increases monotonically with respect to flight hours t_n, and hence the residual strength $R(t_n)$ [Eq. (12)] decreases. The purpose of the periodic inspection is to detect the fatigue cracks. If a fatigue crack is detected, it is repaired and the strength of the component is renewed, thus increasing the structural reliability.

The probability of detecting a fatigue crack in the critical component depends on 1) the probability of inspecting the cracked detail (correct location) in the component and 2) the resolution capability of the crack detection method used for the inspection.[32,36,37] Let U_1 be the probability of inspecting the cracked detail and $U_2(a)$ be the probability of detecting an existing crack of length a when the cracked detail is inspected. U_1 depends on the thoroughness of inspecting all the details and $U_2(a)$ depends on the resolution capability of a particular detection method used for inspection as well as the existing crack size a. Empirical results for the detection probability $U_2(a)$ are available.[32] It is reasonable to assume a minimum crack size a_1 below which the crack cannot be detected and a maximum crack size a_2 beyond which it can certainly be detected. Hence, a possible model for $U_2(a)$ is

$$\begin{aligned} U_2(a) &= 0 && a < a_1 \\ &= [(a-a_1)/(a_2-a_1)]^m && a_1 < a < a_2 \\ &= 1 && a_2 < a \end{aligned} \tag{13}$$

where m is a parameter. It is observed from Ref. 32 that $m = 1$, $a_1 = 0.02$ in., and $a_2 = 0.3$ in. are reasonable approximations for use with the dye penetrant method. Consequently, when a crack of length a exists in the structure, the probability of detecting it, denoted by $F[a]$, is the product of U_1 and $U_2(a)$

$$F[a] = U_1U_2(a) \tag{14}$$

and the probability of not detecting the crack $F^*[a]$ is equal to $1-F[a]$.

IV. Conditional Failure Rate (Risk Function)

As mentioned previously, catastrophic failure occurs as soon as the ultimate strength R_0 [or the residual strength after crack initiation $R(t_n)$] is exceeded by the gust load. It can be observed from Fig. 1 that the problem is a first-passage problem with one-sided threshold.[20–23] The average failure rate (or risk function) per load cycle for the threshold R_0 denoted by $h_0(R_0)$, is therefore[1,22]

$$h_0(R_0) = \nu^+(R_0)/M_c \tag{15}$$

where $\nu^+(R_0)$ is the upcrossing rate given by Eq. (3), and $M_c \geqq 1$ is the average clump size.[1,22] For most structures, particularly for aircraft structures, the threshold R_0 is very high compared to σ_{c2} so that the events of excursion (or exceedance) are statistically independent, and hence $M_c = 1$. We shall set $M_c = 1$ realizing that such an approximation is conservative.[1]

The ultimate strength R_0 for most structures is a random variable.[18,19] For aircraft structures, data has been compiled in Ref. 18 where a Weibull distribution with the shape parameter equal to 19 has been proposed. Therefore, the failure rate h_0 per flight hour follows from Eq. (15) as

$$h_0 = N_0 \int_0^\infty v^+(x) f_{R_0}(x)\,dx \tag{16}$$

where $f_{R_0}(x)$ is the probability density of R_0 and $v^+(x)$ is given by Eq. (3).

Following Ref. 18 that the statistical distribution of the ultimate strength is a Weibull distribution with the shape parameter α_0 and the scale parameter β_0, we obtain the failure rate h_0, by substituting Eq. (3) into Eq. (16) and by making appropriate transformations as follows:

$$h_0 = \sum_{i=1}^{2} P_i N_0 \int_0^\infty e^{-x}\left\{1 - \exp\left[-\left(\frac{x + v_{oi}}{v_{ci}}\right)^{\alpha_0}\right]\right\} dx \tag{17}$$

in which

$$v_{ci} = \beta_0/\sigma_{ci} \qquad i = 1,2 \qquad v_{oi} = X_0/\sigma_{ci} \tag{18}$$

The failure rate h_0 obtained above is the conditional failure rate, the condition being that the fatigue crack has not been initiated.

Let $h(t_n)$ be the failure rate at t_n flight hour after crack initiation; at this time, the residual strength $R(t_n)$ is given by Eq. (12). Then, it can easily be shown that $h(t_n)$ can be computed from Eq. (17) where β_0 appearing in Eq. (18) should be replaced by $\beta_0 \gamma_n$

$$\gamma_n = 1 - (1-\xi)\left(\frac{a(t_n) - a_0}{a_s - a_0}\right)^{1/2} \tag{19}$$

When the residual strength follows Eq. (11), it is obvious that $h(t_n) = h_0$ for $t_n \leq t_c$, where t_c is given by Eq. (10). For $t_n > t_c$, $h(t_n)$ can also be computed from Eq. (17), where β_0 appearing in Eq. (18) should be replaced by $\beta_0 \gamma_n{}^*$ [see Eq. (11)]

$$\gamma_n{}^* = [1 - a_c\, cQ(t_n - t_c)] \tag{20}$$

If the statistical distribution of the ultimate strength R_0 is assumed to be normal with a mean value μ_0 and a coefficient of variation V_0 (dispersion), the failure rate $h(t_n)$ can be obtained in a closed form as follows:

$$h(t_n) = \sum_{i=1}^{2} P_i N_0 \left\{\frac{1}{2} erf\left(\frac{\eta_i}{(2)^{1/2} r_i}\right) + \frac{1}{2}\exp\left[-(2\eta_i - r_i^2)/2\right] \times \left[1 + erf\left(\frac{\eta_i - r_i^2}{(2)^{1/2} r_i}\right)\right]\right\} \tag{21}$$

in which

$$\eta_i = (\gamma_n \mu_0/\sigma_{ci}) - (X_0/\sigma_{ci}) \qquad i = 1,2 \qquad r_i = V_0 \gamma_n \mu_0/\sigma_{ci} \tag{22}$$

where γ_n is given by Eq. (19). The failure rate h_0 before crack initiation can be computed from Eq. (21) where γ_n appearing in Eq. (22) is 1.0. For the case where the residual strength follows Eq. (11), γ_n should be replaced by $\gamma_n{}^*$ [see Eq. (20)].

V. Probability of Failure under Periodic Inspection

Since the time to crack initiation is a random variable, the following formula for the conditional probability will be used frequently

$$P[A] = \int_0^\infty P[A\,|\,t]\,W(t)\,dt \tag{23}$$

where $P[A]$ is the probability of failure and $P[A\,|\,t]$ is the probability of failure under the condition that the crack is initiated at time t. $W(t)$ is the density function of crack initiation given by Eq. (4). Furthermore, if the total failure rate within an interval of time is denoted by K, then the probability of failure P_f in that time interval is

$$P_f = 1 - e^{-K} \tag{24}$$

Let P_0 be the probability of failure within the intended service life T (flight hours) without inspection. Then, it follows from Eqs. (23) and (24) that

$$P_0 = \int_0^T \left[1 - e^{-th_0 - \int_0^{T-t} h(t)dt}\right] W(t)\,dt + \int_T^\infty \left[1 - e^{-Th_0}\right] W(t)\,dt \tag{25}$$

The first term is the probability of failure under the condition that the fatigue crack is initiated within $[0, T]$. The second term represents the probability of failure when the fatigue crack is initiated after the service life T, in which case the total failure rate is Th_0.

Define $H(t_n)$ as the summation of failure rate from the crack initiation to t_n flight hours after crack initiation

$$H(t_n) = \int_0^{t_n} h(t)\,dt \tag{26}$$

Then, with the aid of Eq. (4), P_0 can be written as

$$P_0 = 1 - \exp\left[-Th_0 - (T/\beta)^\alpha\right] - \int_0^T W(t) \times \exp\left[-th_0 - H(T-t)\right] dt \tag{27}$$

Suppose the structure undergoes a periodic inspection at each T_0 flight hours [see Fig. 2]. Let $P_j{}^*$ be the probability of failure in j service intervals $[0, jT_0]$ under the condition that the crack is initiated after $j-1$th inspection. Then, it follows from Eq. (23) that

$$P_j{}^* = \int_0^{T_0} W[(j-1)T_0 + t] \times [\![1 - \exp\{-h_0[(j-1)T_0 + t] - H(T_0 - t)\}]\!]\,dt + \int_{jT_0}^\infty W(t)[1 - \exp(-jT_0 h_0)]\,dt \qquad j = 1, 2, \ldots$$

in which the first term denotes the failure probability in $[0, jT_0]$, when the fatigue crack is initiated in the jth service interval $[(j-1)T_0, T_0]$, and the second term denotes the failure probability when the fatigue crack is initiated after jT_0.

With the aid of Eq. (4), $P_j{}^*$ can be simplified as follows:

$$P_j{}^* = \exp\{-[(j-1)T_0/\beta]^\alpha\} - \exp\{-[jT_0/\beta]^\alpha - jT_0 h_0\} - \int_0^{T_0} W[(j-1)T_0 + t]\exp\{-h_0[(j-1)T_0 + t] - H(T_0 - t)\}\,dt \qquad j = 1, 2, \ldots \tag{28}$$

Let $P(j)$ be the probability of failure within j service intervals $[0, jT_0]$ under periodic inspection. It is obvious that the probability of failure within the first service interval [Fig. 2] $P(1)$ is equal to $P_1{}^*$

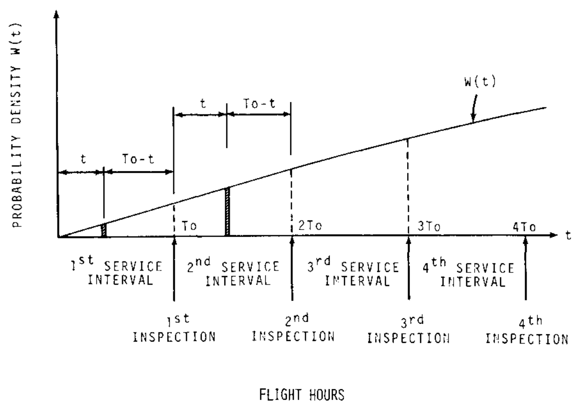

Fig. 2 Periodic inspection, service interval, and probability density for crack initiation.

$$P(1) = P_1^* \tag{29}$$

and the total failure rate in this time interval denoted by K_1, follows from Eqs. (24) and (29)

$$K_1 = -\ln[1-P(1)] \tag{30}$$

The probability of failure in the first two service intervals $[0, 2T_0]$ can be written as

$$P(2) = P_2^* + \int_0^{T_0} q_{12}(t)W(t)\,dt \tag{31}$$

where P_2^* is the contribution from the event of crack initiation after the first inspection given by Eq. (28), and the second term on the right-hand side is the contribution from the event of crack initiation in the first service interval $[0, T_0]$. $q_{12}(t)$ is the failure probability under the condition that the crack is initiated at time t, which consists of two parts

$$q_{12}(t) = F[a(T_0-t)]C_{12}^{(1)}(t) + F^*[a(T_0-t)]V_{12}(t) \tag{32}$$

in which $a(T_0-t)$ is the crack size at the first inspection time T_0. $F[a(T_0-t)]$ is the probability that this crack is detected at T_0 and $F^*[a(T_0-t)] = 1-F[a(T_0-t)]$ is the probability of not detecting the crack at T_0. Both $a(T_0-t)$ and $F[a(T_0-t)]$ are computed from Eqs. (8) and (14), respectively. $V_{12}(t)$ is the failure probability in $[0, 2T_0]$ under the condition that the crack, initiated at time t, is not detected at the first inspection. Hence

$$V_{12}(t) = 1-\exp[-h_0t - H(2T_0-t)] \tag{33}$$

The term $C_{12}^{(1)}(t)$ denotes the failure probability in $[0, 2T_0]$ under the condition that the crack, initiated at time t, is detected at T_0

$$C_{12}^{(1)}(t) = 1-\exp[-h_0t - H(T_0-t) - K_1] \tag{34}$$

where $h_0t + H(T_0-t)$ is the total failure rate in $[0, T_0]$ and K_1 is the (renewal) total failure rate in $[T_0, 2T_0]$, which is the same as the failure rate for $P(1)$ [Eq. (30)], because the crack is detected and the renewal process for the structure occurs after the first inspection.

The probability of failure within $[0, 3T_0]$ can be written as follows:

$$P(3) = P_3^* + \int_0^{T_0} q_{13}(t)W(t)\,dt + \int_0^{T_0} q_{23}(t)W(T_0+t)\,dt \tag{35}$$

in which the second and the third terms are the failure probabilities contributed by the events of crack initiation in the first service interval and in the second service interval, respectively [see Fig. 2].

The failure probability $q_{13}(t)$, under the condition that the crack is initiated at time t (in the first service interval), consists of three parts

$$q_{13}(t) = F[a(T_0-t)]C_{13}^{(1)}(t) + F^*[a(T_0-t)]F[a(2T_0-t)]C_{13}^{(2)}(t) + F^*[a(T_0-t)]F^*[a(2T_0-t)]V_{13}(t) \tag{36}$$

where $a(2T_0-t)$ is the crack size at the second inspection time $2T_0$, when the crack is initiated at time t. Equation (36) is self-explanatory. The first term is the failure probability contributed by the event of crack detection at the first inspection time. The second term is contributed by the event that the crack is not detected by the first inspection but by the second inspection. The third term is contributed by the event that the crack is not detected by both inspections. Hence

$$\begin{aligned} V_{13}(t) &= 1-\exp[-h_0t - H(3T_0-t)] \\ C_{13}^{(2)}(t) &= 1-\exp[-h_0t - H(2T_0-t) - K_1] \\ C_{13}^{(1)}(t) &= 1-\exp[-h_0t - H(T_0-t) - K_2] \end{aligned} \tag{37}$$

where K_2 is the total renewal failure rate in $[T_0, 3T_0]$, which is the same as the total failure rate for $P(2)$

$$K_2 = -\ln[1-P(2)] \tag{38}$$

The failure probability $q_{23}(t)$ [Eq. (35)], under the condition that the crack is initiated at time T_0+t, consists of two parts

$$q_{23}(t) = F[a(T_0-t)]C_{23}^{(1)}(t) + F^*[a(T_0-t)]V_{23}(t) \tag{39}$$

where the first term denotes the failure probability when the crack is detected, and the second term when the crack is not detected at the second inspection time

$$\begin{aligned} V_{23}(t) &= 1-\exp[-h_0(T_0+t) - H(2T_0-t)] \\ C_{23}^{(1)}(t) &= 1-\exp[-h_0(T_0+t) - H(T_0-t) - K_1] \end{aligned} \tag{40}$$

In a similar fashion, the general solution for the probability of failure within j service intervals $[0, jT_0]$ can be obtained recursively as follows:

$$P(j) = P_j^* + \sum_{i=1}^{j-1} \int_0^{T_0} q_{ij}(t)W[(i-1)T_0+t]\,dt \qquad j = 2,3,\ldots \quad i = 1,2,\ldots,j-1$$

$$q_{ij}(t) = F[a(T_0-t)]C_{ij}^{(1)}(t) + \left\{\prod_{k=1}^{j-i} F^*[a(kT_0-t)]\right\}V_{ij}(t) + \delta_{j-i-2}\sum_{k=2}^{j-i}\left\{\prod_{m=1}^{k-1} F^*[a(mT_0-t)]\right\}F[a(kT_0-t)]C_{ij}^{(k)}(t) \tag{41}$$

$$\begin{aligned} V_{ij}(t) &= 1-\exp\{-h_0[(i-1)T_0+t] - H[(j-i+1)T_0-t]\} \\ C_{ij}^{(k)}(t) &= 1-\exp\{-h_0[(i-1)T_0+t] - H(kT_0-t) - K_{j-i-k+1}\} \\ & \qquad k = 1,2,\ldots,(j-i) \\ K_k &= -\ln[1-P(k)] \end{aligned} \tag{41a}$$

where $\delta_{j-i-2} = 1$ if $j-i-2 \geq 0$, and $\delta_{j-i-2} = 0$ otherwise.

The probability of failure derived in Eq. (41) holds for a single airplane. For a fleet of M airplanes, the fleet reliability is defined as the probability of no failure at all.[24,25] Since the event of failure of each airplane is statistically independent, the fleet reliability in j service intervals $[0, jT_0]$, denoted by $R_M(j)$, is $R_M(j) = [1-P(j)]^M$ and the probability of first failure in a fleet of M airplanes is

$$P_M(j) = 1-R_M(j) = 1-[1-P(j)]^M \tag{42}$$

VI. Numerical Example and Computational Procedure

A numerical example is given herein to demonstrate the approach proposed in this study. The parameters associated with the design gust loading are as follows:[7] $P_1 = 99.5\%$, $P_2 = 0.5\%$, $\sigma_{c1} = 0.07g$, $\sigma_{c2} = 0.18g$, where $1g = 10$ ksi [see Eq. (3)]. It is assumed that each flight is of two hours duration and in one flight hour the structure is subjected to 600 load cycles, i.e., $N_0 = 600$, $N_a = 1200$ [see Eq. (8)]. The average fourth power of the ground-air-ground cycle is $\bar{Z}^4 = (1.5g)^4$, and the initial crack size at crack initiation is $a_0 = 0.04$ in. [Eq. (8)]. The shape parameter for crack initiation is $\alpha = 4$ and the scale parameter $\beta = 30{,}000$ hr [Eq. (4)]. The material of the critical component is aluminum. The mean value of the ultimate strength R_0 is $\mu_0 = 5.7g$ and the dispersion is $V_0 = 5.6\%$ [see Eq. (21)]. The critical stress intensity factor $K_c = 75$ ksi (in.)$^{1/2}$. The fail-safe crack size at which the crack is arrested by the crack stoppers $a_s = 7$ in. and the residual strength at a_s is equal to 43% of the ultimate strength; i.e., $\xi = 0.43$ [Eq. (12)]. The thresholds for crack detection are $a_1 = 0.02$ in., $a_2 = 2$ in., and $m = 0.2$ [see Eq. (13)]. Further assume that every detail in the critical component is inspected at the inspection time; i.e., $U_1 = 1.0$ [Eq. (14)]. The crack propagation factor under Gaussian random loading $c = 0.6 \times 10^{-7}$ ksi (in.)$^{1/2}$ is taken from the testing results of Refs. 27–28 [see Eq. (8)]. The design service life for the airplane is $T = 15{,}000$ flight hr. The power spectral density $G^*(\omega)$ of gust load is such that $A = 115$ [Eqs. (A1) and (A2)].

With all the input parameters given above, the computational procedure is summarized as follows: 1) Compute the crack size, $a(t)$, after crack initiation, using Eq. (8). Some results are shown in Fig. 3. 2) Compute the residual strength $R(t)$, after crack initiation, using Eq. (11) or (12). Some results using Eq. (12) are plotted in Fig. 3. 3) Compute the conditional failure rates h_0 and $h(t)$, using either Eqs. (17–20) or Eqs. (21–22). Some results using Eqs. (21) and (22) are plotted in Fig. 3. 4) Compute the cumulative failure rate $H(t)$, using Eq. (26). 5) Compute the detection probability $F[a(t)]$ using Eqs. (13) and (14), where $a(t)$ has been evaluated in the procedure (1). 6) Compute P_j^* using Eq. (28) for $j = 1, 2, \ldots, N$. 7) Compute the failure probability $P(j)$ in $[0, jT_0]$ for $j = 2, \ldots, N$, using Eq. (41).

Results for the first failure probability $P_M(j)$ [Eq. (42)] for a fleet of 50 airplanes as a function of service flight hours are

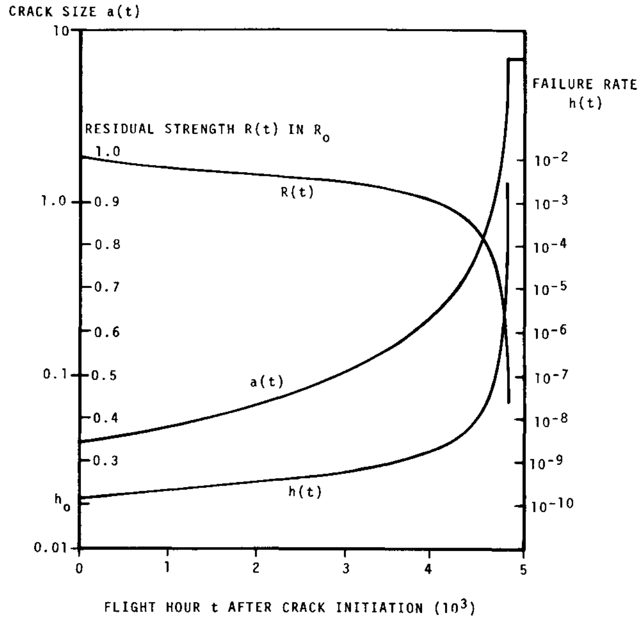

Fig. 3 Crack length $a(t)$, residual strength $R(t)$, and failure rate $h(t)$ after crack initiation.

plotted in Fig. 4 for different number of inspections. The entire computation takes 2 min on a CDC-6600 computer.

It is very interesting to note that the curve for the failure probability under no inspection, $N = 0$, consists of two segments with completely different characteristics. The failure rate in the first segment from 0–5000 flight hr is essentially h_0. This can be visualized from the fact that even though the fatigue crack is initiated at the initial service time $t = 0$, it takes approximately 4700 hr for the crack to reach the fail-safe crack size a_s as clearly shown in Fig. 3. The failure in this region is essentially attributed to the exceedance of the ultimate strength R_0. As a result, inspection in this time interval [0–5000 hr] has little effect in respect to an improvement of the fleet reliability. It can be observed from Fig. 4 that all the curves coincide in this region.

The second segment is in the region from 6000–15,000 flight hr. In this region, the crack initiated in the early service hours has reached its fail-safe crack size and hence failure is essentially attributed to the exceedance of the residual strength ξR_0. Therefore, the failure rate is much higher than h_0 (see Fig. 3). This is a typical characteristic of the progressive fatigue damage effect. Because of the existence of the fatigue crack, the inspection in this region has a significant effect on the fleet reliability, as clearly shown in Fig. 4.

Consequently, inspection at later service time is much more efficient than at the early service time. This, however, is only true if we are confident of the loading, material/structural fatigue performance and the structural analysis. Otherwise, the early service time inspection is still desirable, because it will enable one to discover any deficiencies in the design of the airplane, and to detect if other uncontrollable factors, such as manufacturing variability, corrosion, etc., have a significant effect on the fatigue crack initiation. It is the current practice to perform early inspection so that necessary action, e.g., redesign, can be taken if unexpected fatigue cracks are detected in the early service life.

As mentioned previously, the purpose of inspection is to detect the cracked detail and repair it. Therefore, the ultimate benefit one can achieve through the inspection is to maintain the airplane in a crack-free condition. Under the crack-free condition, the failure rate is h_0 and the failure is traditionally referred to as the chance failure. This ultimate improvement is shown in Fig. 4 by the curve associated with 39 inspections. It can be observed that this curve is practically the extension of the first segment of the curve $N = 0$. As a result, the number of inspections beyond this limit results in no benefit at all. This can be observed from Fig. 4 where the curve associated with 49 inspections practically coincides with the curve associated with 39 inspections.

The probability of first failure P_f in the intended service life of 15,000 flight hours for a fleet of M airplanes is plotted in Fig. 5 as a function of the number of inspections N, for different fleet size M. It indicates clearly the effect of both the inspection and the fleet size on the fleet reliability or the probability of first failure.

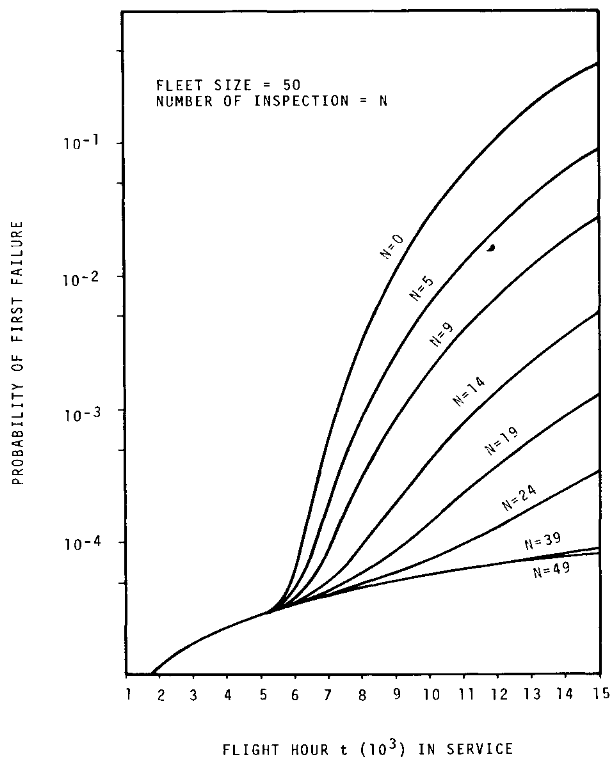

Fig. 4 Probability of first failure in a fleet of 50 airplanes vs service time and number of inspections.

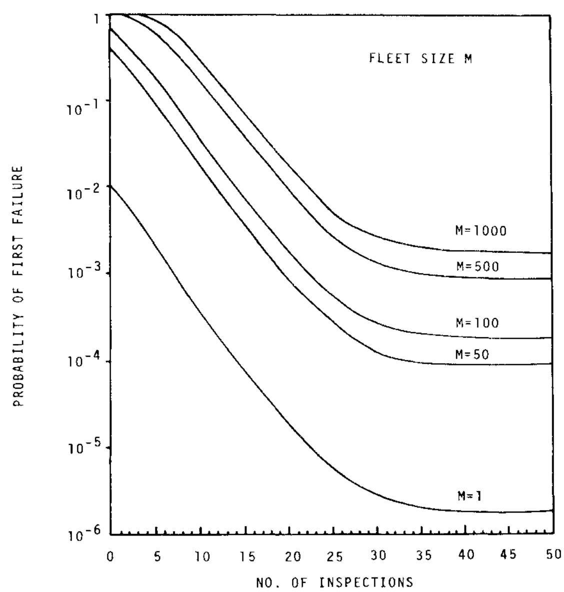

Fig. 5 Probability of first failure vs number of inspections and fleet size.

VII. Discussions and Conclusions

A reliability analysis for fatigue-sensitive aircraft structures based on the application of the approach developed in the random vibration theory, has been presented. In the development of this paper, various assumptions and restrictions have been made which can be relaxed in a more extensive subsequent study. Nevertheless, it is believed that the results presented herein are representative and would not undergo major qualitative changes if these assumptions were relaxed, although quantitative changes would be expected. The assumptions, restrictions, and their implications are discussed as follows.

The first restriction is that the flight-by-flight random loading considered is valid only for the design of transport-type aircraft (e.g., bombers, tankers, high-altitude logistic aircraft, etc.). For fighter aircraft, however, maneuver loading is the major cause of fatigue damage. The occurrence of the maneuver load is a random event and the resulting loading history is a random process. To date, the maneuver loading has not been characterized as a stochastic process as it should be.[41] Data, such as the exceedance curves or peak counts for fighter aircraft has been available, and they exhibit asymmetric characteristics rather than the symmetric characteristics of gust loading given by Eq. (3), where the upcrossing rate is the same as the downcrossing rate. This is a clear indication that the maneuver load is non-Gaussian in nature and warrants a further study for the statistical characterization of such a random process.

The number of load cycles N_0 per flight hour and the number of flight hours per flight, or N_a, are considered as deterministic parameters. These, in principle, should be treated as random variables. Since, however, the average number of load cycles per flight hour is large, the effect of their randomness on the final reliability estimate is believed insignificant.

In predicting the crack propagation under Gaussian random loading [see Eq. (5)], the crack propagation factor c has been considered as a deterministic parameter, because it is believed that its statistical dispersion can be neglected, as is indicated by limited experimental data.[27,28] In fact, the variability of c reflects the statistical variability of the fatigue behavior of materials in response to the random loading. This is because the statistics of the random loading, i.e., $\overline{S^4}$, has been taken into account. It has been observed from an extensive data base that the statistical dispersion of fatigue life under *spectrum loading* is much smaller than that under constant amplitude loading. The dispersion under *random loading* is even less than that under spectrum loading. Although it may be justified to neglect the statistical dispersion of c,[27,28] extensive data is needed for further verification. When c is considered a random variable, the solution for the failure probability can be obtained in exactly the same manner discussed, although the numerical computation for failure rates h_0 and $h(t_n)$ will be quite time consuming.

Both the ultimate strength R_0 and the residual strength $R(t_n)$ after crack initiation have been treated as random variables as they should be.[18] This fact is important and should be emphasized. Our computational experience indicates that there is a significant difference in failure rates, h_0 and $h(t_n)$, and hence in the failure probability, when they are treated as deterministic quantities. The difference in the failure rates, h_0 and $h(t_n)$, ranges from one to two orders of magnitude higher for the case where R_0 and $R(t_n)$ are considered as random variables. As a result, failure rates are very unconservative without treating R_0 and $R(t_n)$ as random variables.

For the sake of simplicity of the presentation, only the failure probability under periodic inspection is derived. The inspection may not be periodic. The solution for failure probability under nonperiodic inspections can be derived easily in a similar fashion as discussed, except that the total renewal failure rates K_j, $j = 1, 2, \ldots$ [see Eq. (41a)] have to be evaluated differently, since Eq. (41a) no longer holds. The evaluation of K_j involves no analytical difficulty.

It has been observed from the numerical example that the nondestructive inspection has a significant effect on the fleet reliability. In particular, the fleet reliability increases as the inspection frequency increases. However, the cost of inspection and maintenance increases also as the frequency of the inspection increases. As a result, there is a tradeoff between the fleet reliability and the cost of maintenance. In this connection, there are a number of variables which can be adjusted in such a way that an objective function or utility can be optimized. Work on this subject will be reported shortly.

Finally, several statistical variables have not been accounted for in the present study, because of the lack of statistical information. Typical examples are 1) the statistical variability in aircraft performance resulting from defects caused in the manufacturing and assembling process, 2) the statistical variability of environmental effects such as stress-corrosion, corrosion fatigue, and buffeting effect, and 3) the probability of making errors in the structural analysis and in loading prediction, resulting from a lack of sufficient information. These random variables should be taken into consideration in the reliability analysis of aircraft structures when their statistical background information becomes available.

Appendix: Statistics of Rise and Fall of Random Processes

The technique proposed in Refs. 26 and 27 for evaluating the statistics of rise and fall of a stationary Gaussian random process $S(t, \sigma_i)$ is rather cumbersome. However, a simpler approximation has been suggested in Ref. 29 as follows:

$$\overline{S^4}(\sigma_i) = A\sigma_i^{\,4} \tag{A1}$$

$$A = 16 + 12\pi\, {}_2F_1(-\tfrac{1}{2}, -\tfrac{3}{2}; 1; k_0^{\,2}) + 24\, {}_2F_1(-1, -1; 1; k_0^{\,2}) \tag{A2}$$

where $\overline{S^4}(\sigma_i)$ is the average of the fourth power of rise and fall of the Gaussian process $S(t, \sigma_i)$ and ${}_2F_1(\cdot)$ is the hypergeometric function,

$$k_0^{\,2} = k_0^{\,2}(\tau) = \rho^2(\tau) + \lambda^2(\tau)$$

$$\lambda^2(\tau) = \int_0^\infty G^*(\omega) \cos \omega\tau \, d\omega \tag{A3}$$

$$\rho^2(\tau) = \int_0^\infty G^*(\omega) \sin \omega\tau \, d\omega$$

$$\tau = \pi/\omega_0 \tag{A4}$$

in which ω_0 is the representative frequency of $S(t)$, and $G^*(\omega)$ is the normalized one-sided power spectral density of $S(t, \sigma_i)$, which is identical for all $i = 1, 2, \ldots$ as described in the text.

Therefore, $\overline{S^4}$ for the composite Gaussian process $S(t)$ can be obtained from Eqs. (2) and (A1) and as follows:

$$\overline{S^4} = \int_0^\infty \overline{S^4}(x) f_\sigma(x)\, dx = 3A(P_1\sigma_{c1}^{\,4} + P_2\sigma_{c2}^{\,4}) \tag{A5}$$

where A is given by Eq. (A2).

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